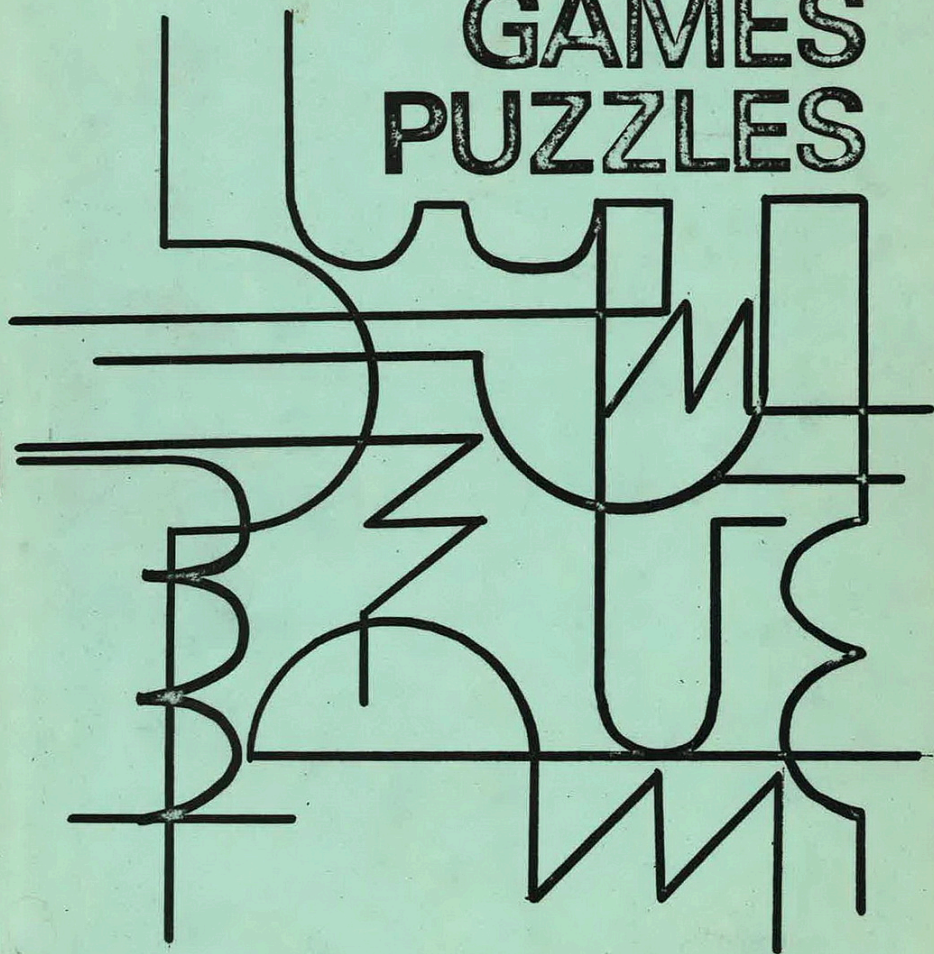


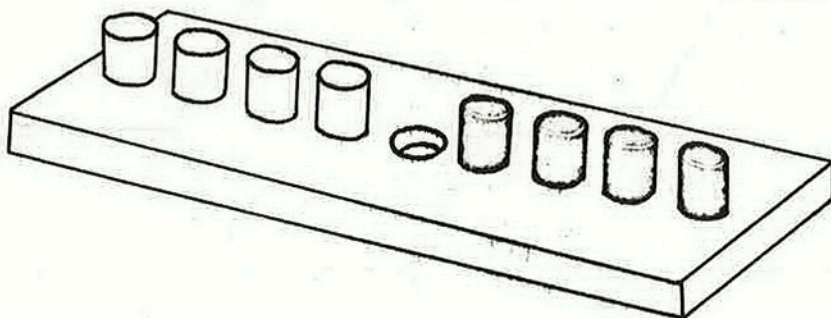
MATHEMATICAL EXPERIMENTS GAMES PUZZLES



MATHEMATICAL ASSOCIATION OF S.A.

JUMP IT.

INTRODUCTION - The solution to this problem demands that the class use their knowledge about systems of linear equations and quadratic equations to discover a new generalization.

MATERIALS NEEDED.

The game consists of a board with a row of nine holes as shown above. Four black pegs are placed in the four holes at one end, and four white pegs are placed in four holes at the other end. There is an empty hole in the middle. A row of two coloured counters could be used instead of the above board.

RULES.

- 1) The object of the game is to exchange the positions of the black and white pegs.
- 2) A peg may be moved from one hole to another empty hole.
- 3) A peg may be moved from one hole to the next one or by "jumping" over at most one peg.
- 4) Students are urged to find a minimal solution using the least number of moves.

QUESTIONS TO ASK.

Can you find an equation relating N , the number of moves, with n the number of black pegs?

Students need to complete the puzzle with different numbers of black pegs to arrive at the generalized formula.

| | | | | | | | | |
|-------------------------|-----|---|---|----|----|----|----|----|
| Number of black pegs | (n) | 1 | 2 | 3 | 4 | 5 | .. | .. |
| Minimum number of moves | (N) | 3 | 8 | 15 | 24 | 33 | .. | .. |

THEORY BEHIND GAME

Constant second order differences indicate the relation is quadratic, or of the form $y = ax^2 + bx + c$.

Since $x = 1$ and $y = 3$, then $3 = a + b + c$.

Since $x = 2$ and $y = 8$, then $8 = 4a + 2b + c$.

Since $x = 3$ and $y = 15$, then $15 = 9a + 3b + c$.

Solving simultaneous equations yields

$$a = 1, \quad b = 2 \quad \text{and} \quad c = 0.$$

Thus, the general formula for this game is

$$y = x^2 + 2x \quad \text{or} \quad x(x + 2).$$

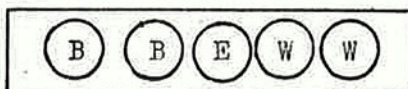
ie. where n = number of black pegs

N = minimum number of moves required

$$N = n(n + 2)$$

EXAMPLE OF GAME.

Below is presented the solution for the case when the number of black pegs is 2 and the number of white pegs is 2.



B = Black peg W = White peg E = Empty hole.

The above arrangement is represented by

B B E W W

Here are the moves.

START

| B | B | E | W | W |
|---|---|---|---|---|
| B | E | B | W | W |
| B | W | B | E | W |
| B | W | B | W | E |
| B | W | E | W | B |
| E | W | B | W | B |
| W | E | B | W | B |
| W | W | B | E | B |
| W | W | E | B | B |